

SOME CONJECTURES ON MODULAR REPRESENTATIONS OF AFFINE \mathfrak{sl}_2 AND VIRASORO ALGEBRA

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ABSTRACT. We conjecture an explicit bound on the prime characteristic of a field, under which the Weyl modules of affine \mathfrak{sl}_2 and the minimal series modules of Virasoro algebra remain irreducible, and Goddard-Kent-Olive coset construction for $\widehat{\mathfrak{sl}}_2$ is valid.

1. INTRODUCTION

This note contains some speculation on modular representations of infinite-dimensional Lie algebras. The modular representations of infinite-dimensional Lie algebras are little understood, and in particular Lusztig-type conjecture (cf. [Lu80]) on irreducible characters in modular representation theory seems to be out of reach in the infinite-dimensional setting for now. We hope our explicit conjectures, though modest, might help to stimulate others to continue further in this challenging new direction.

Let \mathbb{F} be an algebraically closed field of characteristic $p > 2$. We speculate on an explicit bound for the prime characteristic of \mathbb{F} such that the Weyl modules of the affine Lie algebra $\widehat{\mathfrak{sl}}_2$ and the minimal series modules of Virasoro algebra remain irreducible over \mathbb{F} . One remarkable feature is that the two family of modules share the same bound on primes. This leads to another conjecture that the Goddard-Kent-Olive coset construction for $\widehat{\mathfrak{sl}}_2$ is valid under the same bound on $\text{char } \mathbb{F}$.

There are two (a priori unrelated) works [DR13, Lai13] which motivated this note. Lai [Lai13] constructed nontrivial homomorphisms between Weyl modules of $\widehat{\mathfrak{sl}}_2$ at positive integral levels, and showed that the modular representations change dramatically once we go beyond level one and there was no obvious conjectural bound on the prime p for which the Weyl modules remain irreducible. In [Lai13, Table 2], a list of reducible Weyl modules of $\widehat{\mathfrak{sl}}_2$ (together with the lowest level ℓ detected by the method therein relative to a given prime p) is given, in which one sees the prime p can increase rather quickly relative to the level ℓ .

Recall there are 3 minimal series modules of Virasoro algebra of central charge $\frac{1}{2}$ over \mathbb{C} , of highest weight $0, \frac{1}{2}, \frac{1}{16}$, respectively. In [DR13], Dong and Ren showed that the minimal series modules of central charge $\frac{1}{2}$ remain irreducible over \mathbb{F} , if $\text{char } \mathbb{F} \neq 2, 7$. The prime 7 is bad since the values $\frac{1}{2}$ and $\frac{1}{16}$ coincide in \mathbb{F} of characteristic 7.

Our main conjecture is that, under the assumption $\text{char } \mathbb{F} > 2\ell^2 + \ell - 3$ (for each $\ell \in \mathbb{Z}_{\geq 2}$), the Weyl modules of $\widehat{\mathfrak{sl}}_2$ of level ℓ and the minimal series modules of Virasoro algebra of central charge c_ℓ remain irreducible, and the GKO coset construction is valid. This numerical bound $2\ell^2 + \ell - 3$ ensures the highest weights of the minimal series modules of Virasoro algebra of central charge c_ℓ given by (2.1) are distinct in \mathbb{F} (the setting for [DR13] corresponds to $\ell = 2$, with $c_2 = \frac{1}{2}$ and $2\ell^2 + \ell - 3 = 7$). We then check this bound does not contradict with constraints coming from $\widehat{\mathfrak{sl}}_2$ in [Lai13].

2. THE CONJECTURES

2.1. On irreducibility of Virasoro minimal series. Let \mathbb{F} be an algebraically closed field of characteristic $p > 2$. Recall that the Virasoro algebra is the Lie algebra over \mathbb{F} , $\text{Vir} = \mathbb{F}C \oplus \bigoplus_{n \in \mathbb{Z}} \mathbb{F}L_n$, subject to the commutation relations: C is central, and

$$[L_m, L_n] = (m - n)L_{m+n} + \delta_{m+n,0} \frac{1}{2} \binom{m+1}{3} C, \quad (m, n \in \mathbb{Z}).$$

Set $\text{Vir}_+ = \bigoplus_{n=1}^{\infty} \mathbb{F}L_n$, $\text{Vir}_- = \bigoplus_{n=1}^{\infty} \mathbb{F}L_{-n}$. Given $c, h \in \mathbb{F}$, the Verma module $M_{c,h}$ over Vir is a free $U(\text{Vir}_-)$ -module generated by 1, such that $\text{Vir}_+ \cdot 1 = 0$, $L_0 \cdot 1 = h1$ and $C \cdot 1 = c1$. Denote by $L_{c,h}$ the unique irreducible quotient Vir -module of $M_{c,h}$. The scalar c is called the central charge of $L_{c,h}$.

For $\ell \in \mathbb{Z}_{\geq 2}$, set

$$(2.1) \quad c_\ell = 1 - \frac{6}{(\ell+1)(\ell+2)}.$$

Note $c_2 = \frac{1}{2}$, $c_3 = \frac{7}{10}$, $c_4 = \frac{4}{5}$. For $1 \leq m \leq \ell$, $1 \leq n \leq \ell+1$, we let

$$(2.2) \quad h_{m,n} = h_{m,n;\ell} = \frac{(m(\ell+2) - n(\ell+1))^2 - 1}{4(\ell+1)(\ell+2)}.$$

The scalars c_ℓ and $h_{m,n}$ in \mathbb{F} are understood after canceling out common integer factors in the numerators and denominators, and so it is possible that these scalars are well defined even when $\text{char } \mathbb{F}$ divides $\ell+1$ or $\ell+2$. (For example, $c_2 = 1 - \frac{6}{12} = \frac{1}{2} \in \mathbb{F}$ makes sense in characteristic $p = 3$.) Note that $h_{m,n} = h_{\ell+1-m, \ell+2-n}$. To avoid such a double counting, it is well known that one can simply impose the constraint $n \leq m$. The Vir -modules $L_{c_\ell, h_{m,n}}$ ($1 \leq n \leq m \leq \ell$) are usually referred to as the (unitary) *minimal series*.

Conjecture 1. Let $\ell \in \mathbb{Z}_{\geq 2}$. Assume $\text{char } \mathbb{F} > 2\ell^2 + \ell - 3$.

- (1) The minimal series Vir -modules $L_{c_\ell, h_{m,n}}$ (for $1 \leq n \leq m \leq \ell$) over \mathbb{F} are irreducible.
- (2) Let L, L' be minimal series Vir -modules over \mathbb{F} of the same central charge c_ℓ . Then $\text{Ext}^1(L, L') = 0$.

One can further expect that the main theorem of [W93] remain valid over \mathbb{F} under the assumption that $\text{char } \mathbb{F} > 2\ell^2 + \ell - 3$; that is, the Virasoro vertex algebra $L_{c_\ell, 0}$ is rational and has the same fusion rule as in characteristic zero. (This is known to hold for $c_2 = \frac{1}{2}$ by the work of Dong-Ren.)

2.2. On irreducibility of Weyl modules for $\widehat{\mathfrak{sl}}_2$. For basics on modular representations of affine Kac-Moody algebra $\widehat{\mathfrak{g}}$, we refer to Mathieu [Ma96]; also cf. [AW15, Lai13]. The level one Weyl modules of $\widehat{\mathfrak{g}}$ has been shown to be irreducible under the assumption that $p \geq h$ (the Coxeter number) by various authors (deConcini-Kac-Kazhdan, Chari-Jing, Brundan-Kleshchev) in different ways (though all these approaches are built on the fact that the level one Weyl modules afford an explicit combinatorial/vertex operator realization); cf. [Lai13] for references.

Denote by X_ℓ^+ the set of dominant integral weights of level $\ell \in \mathbb{Z}_{\geq 1}$. Note that $X_1^+ = \{\omega_0, \omega_1\}$ consists of 2 fundamental weights. Then $X_\ell^+ = \{\lambda_{\ell,n} = (\ell-n)\omega_0 + n\omega_1 \mid 0 \leq n \leq \ell\}$.

We can define the Weyl module $V(\lambda)$, for $\lambda \in X_\ell^+$ (for $\ell \in \mathbb{Z}_{>0}$) of the affine Lie algebra $\widehat{\mathfrak{sl}}_2$ over \mathbb{F} as in [Ma96].

Conjecture 2. *Let $\ell \in \mathbb{Z}_{>0}$. Assume $\text{char } \mathbb{F} > 2\ell^2 + \ell - 3$.*

- (1) *The Weyl modules $V(\lambda)$ of $\widehat{\mathfrak{sl}}_2$ over \mathbb{F} are irreducible, for $\lambda \in X_\ell^+$.*
- (2) *Let $\lambda, \mu \in X_\ell^+$. Then $\text{Ext}^1(V(\lambda), V(\mu)) = 0$.*

One can rephrase Conjecture 2 as that the category of rational representations of level ℓ of the Kac-Moody group (associated to $\widehat{\mathfrak{sl}}_2$) over \mathbb{F} is semisimple if $\text{char } \mathbb{F} > 2\ell^2 + \ell - 3$.

One can further hope that under the assumption that $\text{char } \mathbb{F} > 2\ell^2 + \ell - 3$ the affine vertex algebra $V(\ell\omega_0)$ is rational and has the same fusion rule as over the complex field \mathbb{C} (which was computed by I. Frenkel and Y. Zhu).

2.3. Modular GKO coset construction. Let $\ell \geq 2$. Recall the Goddard-Kent-Olive ($\widehat{\mathfrak{sl}}_2|_{\ell-1} \oplus \widehat{\mathfrak{sl}}_2|_1, \widehat{\mathfrak{sl}}_2|_\ell$)-coset construction over \mathbb{C} [GKO86] refers to the following tensor product decomposition into a direct sum of multiplicity-free irreducible $(\widehat{\mathfrak{sl}}_2|_\ell, \text{Vir})$ -modules:

$$(2.3) \quad V(\lambda_{\ell-1;n}) \otimes V(\omega_\epsilon) = \bigoplus_{\substack{0 \leq j \leq n \\ j \equiv n+\epsilon \pmod{2}}} V(\lambda_{\ell;j}) \otimes L_{c_\ell, h_{n+1, j+1}} \bigoplus_{\substack{n+1 \leq j \leq \ell \\ j \equiv n+\epsilon \pmod{2}}} V(\lambda_{\ell;j}) \otimes L_{c_\ell, h_{\ell-n, \ell+1-j}}$$

for all $0 \leq n \leq \ell - 1$ and $\epsilon \in \{0, 1\}$.

Conjecture 3 (Modular GKO conjecture). *The multiplicity-free decomposition (2.3) into a direct sum of irreducible $(\widehat{\mathfrak{sl}}_2|_\ell, \text{Vir})$ -module is valid over \mathbb{F} , if $\text{char } \mathbb{F} > 2\ell^2 + \ell - 3$.*

Remark 4. The following “partial semisimple tensor product” statement is a consequence of Conjectures 2 and 3: *Assume $\text{char } \mathbb{F} > 2\ell^2 + \ell - 3$. For any positive integers ℓ_1, ℓ_2 such that $\ell_1 + \ell_2 \leq \ell$ and any $\lambda \in X_{\ell_1}^+, \mu \in X_{\ell_2}^+$, $V(\lambda) \otimes V(\mu)$ is a semisimple $\widehat{\mathfrak{sl}}_2$ -module.*

3. EVIDENCE AND GENERALIZATIONS

3.1. Supporting evidence for Conjecture 1. Recall $h_{m,n}$ from (2.2). A prime p is called **Vir** $|_{c_\ell}$ -**good** if the scalars $h_{m,n}$ ($1 \leq n \leq m \leq \ell$) are pairwise distinct (and hence there are $\ell(\ell+1)/2$ distinct values of such $h_{m,n}$); otherwise, a prime p is called **Vir** $|_{c_\ell}$ -**bad**.

Proposition 5. *Every prime greater than $2\ell^2 + \ell - 3$ are $\text{Vir}|_{c_\ell}$ -good. Equivalently, every $\text{Vir}|_{c_\ell}$ -bad prime does not exceed $2\ell^2 + \ell - 3$.*

Note $2\ell^2 + \ell - 3 = (2\ell + 3)(\ell - 1)$ is a prime only when $\ell = 2$.

Proof. Let $1 \leq m, m' \leq \ell, 1 \leq n, n' \leq \ell + 1$. Denote the numerator of $h_{m,n}$ by

$$\tilde{h}_{m,n} = (m(\ell + 2) - n(\ell + 1))^2 - 1,$$

and denote

$$(3.1) \quad D_{m,n;m',n'}^{\ell,\pm} = (m \pm m')(\ell + 2) - (n \pm n')(\ell + 1).$$

Then

$$\tilde{h}_{m,n} - \tilde{h}_{m',n'} = D_{m,n;m',n'}^{\ell,+} D_{m,n;m',n'}^{\ell,-}.$$

Hence $\tilde{h}_{m,n} = \tilde{h}_{m',n'}$ if and only if $\left((i) D_{m,n;m',n'}^{\ell,+} = 0 \text{ or } (ii) D_{m,n;m',n'}^{\ell,-} = 0 \right)$. Let us consider (i) and (ii) as equations over \mathbb{Z} for now. Assume (i) holds. Since $\{\ell+1, \ell+2\}$ are relatively prime, we have $(\ell+2) \mid (n+n')$ and $(\ell+1) \mid (m+m')$, which further imply that $n+n' = \ell+2$ and $m+m' = \ell+1$, respectively (recall $m, m' \leq \ell$, and $n, n' \leq \ell+1$). Similarly, (ii) holds imply $m = m'$ and $n = n'$.

Denote

$$(3.2) \quad B_\ell = \left\{ |D_{m,n;m',n'}^{\ell,+}| \mid 1 \leq m, m' \leq \ell, 1 \leq n, n' \leq \ell+1 \right\}.$$

(A set defined using $|D_{m,n;m',n'}^{\ell,-}|$ instead is equivalent to B_ℓ thanks to $\tilde{h}_{m,n} = \tilde{h}_{\ell+1-m, \ell+2-n}$.) Note the maximal value in B_ℓ is achieved at $(\ell+\ell)(\ell+2) - (1+1)(\ell+1) = 2(\ell^2 + \ell - 1)$ which is manifestly even; the second largest absolute value is achieved at $(\ell+\ell)(\ell+2) - (1+2)(\ell+1) = 2\ell^2 + \ell - 3$; so under the assumption $\text{char } \mathbb{F} > 2\ell^2 + \ell - 3$, all values of $\tilde{h}_{m,n}$ for $1 \leq n \leq m \leq \ell$ are distinct. \square

Conjecture 1 is known to hold when $c_2 = \frac{1}{2}$ (i.e., $\ell = 2$) [DR13]. A basic observation of [DR13] can be rephrased that the $\text{Vir}|_{c_2}$ -bad primes are $\{2, 7\}$. Note that the $\text{Vir}|_{c_2}$ -good primes $p = 3, 5$ are not detected by Proposition 5; see Remark 9 below for an explanation of these missing primes.

3.2. Supporting evidence for Conjecture 2. [Lai13, Table 2] provides a list of reducible Weyl modules (detected by the approach therein) of lowest levels ℓ at a given prime p . Table 1 below is a somewhat novel look at the data provided [Lai13, Table 2], and it indicates the maximal known prime p for which there exists a reducible Weyl module at level $\ell \geq 2$.

TABLE 1. Maximal known primes p for reducible Weyl modules at level ℓ

ℓ	2	3	4	5	6	7	8
p	3	13	11	23	37	47	53
$2\ell^2 + \ell - 3$	7	18	33	52	75	102	133

We note that $p < 2\ell^2 + \ell - 3$ for all ℓ in the table, and so Conjecture 2 is consistent with the results of [Lai13].

Remark 6. Lai and the author have formulated a (conjectural) linkage principle; see [Lai13, Conjecture 6.1]. If one can show that the weights in X_ℓ^+ are minimal in the Bruhat order in each linkage class assuming $\text{char } \mathbb{F} > 2\ell^2 + \ell - 3$, then Conjecture 2 would follow (modulo the linkage principle conjecture).

3.3. Supporting evidence for Conjecture 3. The evidence from Table 1 (based on [Lai13, Table 2]) for affine algebra $\widehat{\mathfrak{sl}}_2$ is remarkably consistent with the constraints from Proposition 5 for Virasoro algebra. Conjecture 3 offers a reasonable and conceptual way of explaining such a coincidence, and it helps to relate Conjecture 2 and Conjecture 1.

3.4. More precise bound on char \mathbb{F} for Virasoro algebra. One could make a conjecture (which strengthens Conjecture 1) that whenever char \mathbb{F} is a $\text{Vir}|_{c_\ell}$ -good prime the minimal series $L_{c_\ell, h_{m,n}}$ are irreducible. One drawback of this stronger conjectural bound of char \mathbb{F} is that a precise description of the set of $\text{Vir}|_{c_\ell}$ -bad primes is difficult for general $\ell \geq 2$, in contrast to the explicit though coarser bound in Proposition 5. We will provide some partial answer below. The $\text{Vir}|_{c_\ell}$ -bad primes for small values of ℓ can be computed by hand.

Example 7. The $\text{Vir}|_{c_2}$ -bad primes are $\{2, 7\}$ (i.e., the primes $\leq 2\ell^2 + \ell - 3 = 7$ except 3, 5).
 The $\text{Vir}|_{c_3}$ -bad primes are $\{2, 3, 7, 9, 13, 17\}$ (i.e., primes $\leq 2\ell^2 + \ell - 3 = 18$ except 5, 11).
 The $\text{Vir}|_{c_4}$ -bad primes are $\{\text{all primes } \leq 2\ell^2 + \ell - 3 = 33\} \setminus \{5, 19, 29, 31\}$.
 The $\text{Vir}|_{c_5}$ -bad primes are $\{\text{all primes } \leq 2\ell^2 + \ell - 3 = 52\} \setminus \{7, 29, 41, 43, 47\}$.
 The $\text{Vir}|_{c_6}$ -bad primes are $\{\text{all primes } \leq 2\ell^2 + \ell - 3 = 75\} \setminus \{7, 41, 71, 73\}$.

For integers a, b with $a \leq b$, denote by $[a, b]$ the interval of integers between a and b . Set

$$B_\ell(a) = [\ell^2 + \ell + a(\ell + 2), \ell^2 + 2\ell - 1 + a(\ell + 1)], \quad \text{for } 0 \leq a \leq \ell - 1.$$

Note $k < k'$ for all $k \in B_\ell(a), k' \in B_\ell(a')$ whenever $a < a'$.

Proposition 8. *The set B_ℓ (3.2) is given by*

$$B_\ell = [1, \ell^2 + \ell - 2] \bigcup B_\ell(0) \bigcup B_\ell(1) \bigcup \cdots \bigcup B_\ell(\ell - 1).$$

Proof. Recall B_ℓ is defined using $|D_{m,n;m',n'}^{\ell,+}|$; cf. (3.1). We first list the values of $(n+n')(\ell+1)$, for $1 \leq n, n' \leq \ell+1$, in an increasing row (there are $(2\ell+1)$ entries), and list the values of $(m+m')(\ell+2)$, for $1 \leq m, m' \leq \ell$, in an increasing column (there are $(2\ell-1)$ entries). By taking the absolute value of the difference of row and column entries, one produces a $(2\ell-1) \times (2\ell+1)$ matrix A whose entries are given by $|D_{m,n;m',n'}^{\ell,+}| = |(m+m')(\ell+2) - (n+n')(\ell+1)|$. One observes that the matrix is symmetric under rotation by 180 degrees so we only need to consider the $(2\ell-1) \times (\ell+1)$ submatrix, denoted by D , which consists of the $(\ell+1)$ columns of A . By listing the entries at the r th diagonals of D in the following order: $r = 1, 0, 2, -1, 3, -2, \dots, 2-\ell, \ell$, one obtains exactly the interval $[1, \ell^2 + \ell - 2]$. (This is not so miraculous by noting the following: the values in each diagonal form a natural sequence, the last column of D is symmetric by flipping.) Now we are left with the lower ℓ diagonals of D , whose values are given by the ℓ intervals $B_\ell(a)$, for $0 \leq a \leq \ell - 1$.

The easiest way for a reader to convince herself/himself of the above proof is to work out an example for one particular ℓ . For example let $\ell = 5$. Following the recipe in the proof above, we obtain the initial row and column vectors to be $(12, 18, 24, 30, 36, 42, \dots, 72)$ and $(14, 21, 28, 35, 42, 49, 56, 63, 70)^t$. This leads to the following matrix

$$D = \begin{bmatrix} 2 & 4 & 10 & 16 & 22 & 28 \\ 9 & 3 & 3 & 9 & 15 & 21 \\ 16 & 10 & 4 & 2 & 8 & 14 \\ 23 & 17 & 11 & 5 & 1 & 7 \\ 30 & 24 & 18 & 12 & 6 & 0 \\ 37 & 31 & 25 & 19 & 13 & 7 \\ 44 & 38 & 32 & 26 & 20 & 14 \\ 51 & 45 & 39 & 33 & 27 & 21 \\ 58 & 52 & 46 & 40 & 34 & 28 \end{bmatrix}.$$

Then one sees clearly that the above recipe leads to the statement in the proposition. \square

Note that $B_\ell(\ell - 1) = \{2(\ell^2 + \ell - 1)\}$ consists of the largest integer in B_ℓ ; moreover $2\ell^2 + \ell - 3 \in B_\ell(\ell - 2)$ is the largest integer in $B'_\ell := B_\ell \setminus B_\ell(\ell - 1)$, or the second largest integer in B_ℓ . By definition of B_ℓ and Proposition 5, we have

$$(3.3) \quad \{\text{Vir}|_{c_\ell}\text{-bad primes}\} \subseteq [1, 2\ell^2 + \ell - 3] \cap B_\ell.$$

One may regard (3.3) as a sharper form of the description of the bound in Proposition 5, thanks to the concrete description of the set B_ℓ in Proposition 8.

Remark 9. Note $(\ell + 1)^2 \notin B_\ell$ and $(\ell + 2)^2 \notin B_\ell$. Recall the denominator for $h_{m,n}$ is $4(\ell + 1)(\ell + 2)$. It follows that if either $\ell + 1$ or $\ell + 2$ happens to be a prime it must be a $\text{Vir}|_{c_\ell}$ -good prime. In the above examples for small ℓ , this prime happens to be the smallest $\text{Vir}|_{c_\ell}$ -good prime, and the second $\text{Vir}|_{c_\ell}$ -good prime happens to be $\ell^2 + \ell - 1$ (where $\ell^2 + \ell - 1$ happens to be a prime).

Introduce $G_\ell = [1, 2\ell^2 + \ell - 3] \setminus B_\ell$. It follows by definition that the set of $\text{Vir}|_{c_\ell}$ -good primes $\leq 2\ell^2 + \ell - 3$ is contained in G_ℓ (recall all primes $> 2\ell^2 + \ell - 3$ are $\text{Vir}|_{c_\ell}$ -good by Proposition 5). Set

$$G_\ell(a) = [\ell^2 + \ell - 1 + a(\ell + 1), \ell^2 + \ell - 1 + a(\ell + 2)], \quad \text{for } 0 \leq a \leq \ell - 1.$$

Note $k < k'$ for all $k \in G_\ell(a), k' \in G_\ell(a')$ whenever $a < a'$. Then one derives by definition and Proposition 8 that

$$G_\ell = G_\ell(0) \bigcup G_\ell(1) \bigcup \cdots \bigcup G_\ell(\ell - 1).$$

3.5. Higher ranks. It would be interesting to generalize the conjectures of this paper to affine algebras of higher ranks and W-algebras (in particular for $\widehat{\mathfrak{sl}}_n$ and the corresponding W-algebra W_n). One could try to find a conjectural bound on $\text{char } \mathbb{F}$ under which the Weyl modules are irreducible, by analyzing the highest weights of the unitary minimal series of the W-algebras [FKW92]. One can see by a similar method as in Proposition 5 this bound should be some quadratic polynomial on the level and the rank. One can also derive some more evidence from [Lai13, Theorem 5.10] on $\text{char } \mathbb{F}$ for reducible Weyl modules of affine Lie algebras of higher rank (say, $\widehat{\mathfrak{sl}}_3$). Then one would check if the bound arising from W-algebra minimal series is compatible with the bound from affine Lie algebras.

3.6. Other generalizations. Instead of Virasoro algebra, one can consider the super-Virasoro algebra, also known as Neveu-Schwarz (and Ramond) algebras, and its unitary minimal series. A similar analysis can lead to a conjectural bound on $\text{char } \mathbb{F}$ under which the minimal series of the super-Virasoro algebra are irreducible. In the same way that affine Lie algebra $\widehat{\mathfrak{sl}}_2$ is related to Virasoro algebra via the GKO construction, the Neveu-Schwarz algebra is related to the affine Lie superalgebra $\widehat{\mathfrak{osp}}_{1|2}$. So we can give a conjectural bound on $\text{char } \mathbb{F}$ (relative to the levels) under which the Weyl modules of $\widehat{\mathfrak{osp}}_{1|2}$ are irreducible.

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